

Summer HSSP Lecture Notes Week 1

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July 6, 2014

First Class: proofs and friends

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Chapter 1

Glossary of symbols

Below is a list of symbols that will be used to varying degrees in this class. We encourage you to feel free to use them, but won't require them. In addition, if some symbol appears that you don't understand, ask immediately for the english meaning— most of the time its just a simple sentence or idea contracted into a single symbol.

\forall : for all or for each

\exists : there exists

\in : is an element of

\notin : is not an element of

\Rightarrow : implies or leads to

\subset : is a subset of

Σ : summation notation (see induction chapter)

Π : similar to Σ except all additions are replaced by multiplications

\equiv : congruent to (used in modular operations)

\rightarrow : immediate logic flow

\sharp : cardinality

Chapter 2

Types of numbers

There are a variety of different ways to categorize numbers, below we set out the principle divisions. Aside from this, a couple of special subsets are pulled out and briefly examined.

2.1 Breaking it down

All numbers fall into the following simple categorizations:

2.1.1 Number categories

Natural numbers: “Numbers beginning with 1 and 2, where each number is exactly 1 greater than its predecessor.”

Whole numbers: “All the natural numbers, and zero”

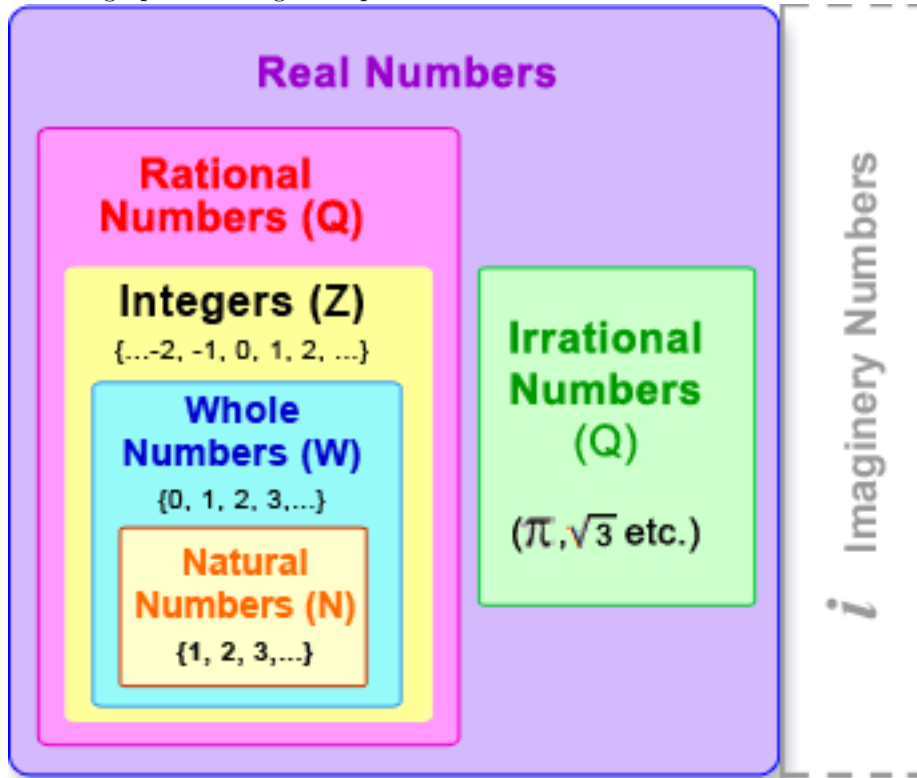
Integers: “The set of all Whole numbers and their additive inverses”

Rational numbers: “All numbers which can be represented as a ratio $\frac{p}{q}$, where p and q are relatively prime integers”

Real numbers: “All numbers without imaginary components” alternatively: “The set containing all rational and irrational numbers”

Irrational numbers: “All numbers which part are part of the Real numbers but not part of the Rational numbers”

The graphic below gives a picture of how these sets of numbers are related:



2.1.2 Primes

Another often considered categorization of numbers is the set of prime numbers. Primes are the set of natural numbers which have the property that their only two factors are 1 and itself. Please note that 1 is not prime, nor is it composite. Occasionally the additive inverse of primes are also considered primes, however in this course we'll focus on positive primes. If you have not learned about primes, learn this definition well; primes will come up repeatedly!

Relatively Prime Numbers

Two numbers are relatively prime if neither number has a common prime factor with the other, aside from 1. The two numbers are usually both natural numbers.

Don't worry too much about primes at this point, this will be covered more next week and throughout the rest of this course.

Chapter 3

Direct Proofs

Proofs are a way to make a mathematical statement that is without a doubt true. Often times the proof doesn't create a formula or theorem, but rather verify its validity. Here we mention direct proofs. This is the most classical way of proving things.

Directly proving things involves using definitions, postulates, theorems, and other already proven or assumed things, in order to prove that something else is true (either a particular example or a lemma or something of that sort). Those of you who've taken geometry should already be familiar with this method of proving things. Geometry proofs involve using a handful of postulates (assumptions that cannot be proven), for instance, in order to prove something about an angle value in a figure.

Technically, most algebra proofs also involve using postulates, but most are not explicitly mentioned; for instance, when multiplying both sides of an equation, you might state, "Assumption of equality means that all multiples of the equality will also be true", but that seems unnecessary since its a very intuitive fact.

3.1 Examples

Now we give a couple of simple examples. In these examples, the standard set of equality postulates include: addition, subtraction, multiplication, division, substitution, and transitive property.

3.1.1 Example 1

Using the standard set of equality postulates, prove the value of x such that:

$$3x+2=8$$

This can be done in an instant using the algebra we know, $x=2$. But now we'll prove this answer using the postulates of equality:

$$\begin{aligned}3x+2&=8, \text{ given} \\3x&=6, \text{ by subtraction postulate (-2 to both sides)} \\x&=2, \text{ by division postulate (division by 3 to both sides)}\end{aligned}$$

Although we (likely) applied the exact same steps in finding x quickly, in this case we've shown work and reasoning as to why this value of x is true without a doubt.

3.1.2 Example 2

Again, using the standard set of equality postulates, prove the values of x and y such that:

$$\begin{aligned}x+4y&=9 \\2y&=4\end{aligned}$$

We can see rather quickly see that $y=2$, and so $x=1$. But we need to show how we derived this conclusion using postulates.

$$\begin{aligned}2y&=4, \text{ given} \\y&=2, \text{ division postulate (division by 2)} \\x+4y&=9, \text{ given} \\x+4(2)&=9, \text{ substitution postulate (y=2)} \\x&=1, \text{ subtraction postulate (-8 to both sides)}\end{aligned}$$

Now we have shown without a doubt that $x=1$ and $y=2$. Although there were slightly different ways to prove this, they all lead to the same answer and have no room for ambiguity. The values have been proven.

As might be guessed, there are more challenging proofs than those given above. In addition, often times there will be no limit on the postulates and theorems that can be used to prove something.

3.2 Questions

Prove the values that solve the following using the standard set of equality postulates:

(*)1. $6x+2=3x+5$

(*)2. $2x+3y=2$ and $2x+4y=4$

(**)3. $12x+2y=6$ and $15x+3z=9$ and $4y+5z=-71$

Chapter 4

Proof by Counter-example

We now begin our journey into the land of proofs. The first type of proof we explore is proof by counter-example. This isn't a very utilized proof method, but it is still effective where applicable. The good news is that this proof method is very straight forward.

The idea is as follows: I state the following:

Proposition: X is always true.

I believe that this proposition is always true and that no exceptions exist, so by showing merely a single exception, you will prove that my proposition is false or at least does not hold for all the cases I claimed it did.

That's all there is to a proof by counter-example.

4.1 Examples

Now we show a couple of examples.

4.1.1 Example 1

Proposition: All natural numbers are even.

Dis-proof: Even numbers are divisible by 2. 1 is a natural number, and one is not divisible by 2, therefore that proposition is not always true.

4.1.2 Example 2

Proposition: There exist no triplet primes, where triplet primes are three prime numbers: a, b, c with $c - b = b - a = 2$.

Dat-proof: Consider the three numbers: 3,5,7. All three are prime numbers, and $7-5=5-3=2$, which breaks that proposition. This is our counter-example to disprove their proposition.

Hopefully these two examples have given you an idea as to how these proofs are done.

4.2 Questions

Find appropriate counter-examples where possible (not all are possible to find counter-examples for):

(*)1. Proposition: In the interval $[0,1)$ there are more natural numbers than irrational numbers.

(*)2. Proposition: $x^2 + 4x + 5 = 0$ has no real values of x which satisfy the equation.

(*)3. Proposition: Everyone in your immediate family is left-handed.

(**)4. Proposition: There are no other triplet primes than 3,5,7.

(**)5. Proposition: define a perfect number as a number whose sum of all factors (prime and composite) are equal to twice its value. For example, 6 has factors 1, 2, 3, 6, which sum to 12. We also notice that 28 is a perfect number. I propose that there are no other perfect numbers than 6 and 28.

(*****)6. Proposition: define a perfect number as above. I propose that there are no odd perfect numbers.

Chapter 5

Proof by induction

Here we introduce our first proof method. I kinda lied by saying that last chapter was the first proof method. Proof by counter-example is more of a disproving method than a proof technique. As we begin true proofs, please take careful (careful!) note that these are proof methods, NOT a way to find solutions to questions, but rather a way to verify the validity of a solution. Lastly, throughout these proofs, only ONE side of the proposed formula can be manipulated—you cannot add something to both sides or divide both sides by something, these operations assume equality.

5.1 A Motivating Example

We begin by considering the following challenge of finding the following sum $1 + 2 + 3 + 4 + \dots + 10 + 11 + \dots + (n - 2) + (n - 1) + n = S(n)$, where the \dots indicates that terms have been visually left out, but that the logical terms will fall in place. Also, notice that S is a function of n , the number of terms we are summing together. Clearly one way to approach this is to simply add together numbers manually, but that's not very quick, nor does it give a formula to immediately generate the sum. However, perhaps we can cleverly solve this challenge with some sort of trick. We begin by noting that we can arrange these numbers in any order when adding them up. I figure that perhaps it would be more convenient to consider the sum as the following two orderings:

$$\begin{aligned} S &= 1 + 2 + 3 + 4 + \dots + 10 + 11 + \dots + (n - 3) + (n - 2) + (n - 1) + n \\ S &= n + (n - 1) + (n - 2) + (n - 3) + \dots + (n - 9) + (n - 10) + \dots + 4 + 3 + 2 + 1 \end{aligned}$$

Adding these together, gives:

$$2S = (n + 1) + (n + 1) + (n + 1) + (n + 1) + \dots + (n + 1) + (n + 1) + \dots + (n + 1)$$

We notice that there are n of these terms. Combining these terms, we have:

$$2S = n * (n + 1), \text{ or simply } S(n) = \frac{n(n+1)}{2}.$$

This is wholly correct, and there is nothing wrong with this answer. How-

ever, later we'll show the validity of this equation not only through the algebra that we did above, but also through our new proof method of induction.

5.2 Sigma Notation

We now introduce a bit of notation. The so-called 'Sigma notation': $\sum_{i=0}^n f(i) = f(0) + f(1) + f(2) + f(3) + \dots + f(n-1) + f(n)$. The letter Sigma is used because it is the Greek letter S, which is the first letter of Summation, which is what Sigma notation signifies. Take notice that there are multiple parts to the Sigma notation: there is a lower bound for the summation, there is an upper bound for the summation, and lastly there is a function of the increasing index (in this case i).

For example, the first example of $1+2+3+\dots+n$ could've been written as: $\sum_{i=1}^n i$

5.2.1 Discrete Numbers Only plz

Please take note that everywhere in this, i and n are both Natural or Whole numbers. This means that i and n can only take the values of $0, 1, 2, 3, 4, 5, 6, 7, \dots, 20, 21, 22, \dots$ —but no numbers with decimals or negative values. This is what typically distinguishes topics in Discrete Mathematics from other branches of math—you can only have countable values.

Just as a side note, it would be possible to do these summations using negative or even rational numbers, however, those summations will not be covered. To do this, you'd merely either specify the separation of each index or you'd divide each index by some factor so that it'd become a rational number rather than a natural number.

5.3 Induction

Now, we finally introduce induction. A standard explanation of induction is the following. Imagine a row of dominoes. The concept is that if I can find the first domino, and guarantee that each domino has another domino which it will cause to fall down, then I know for certain that I can knock down the entire chain of dominoes. In a mathematical setting, this is rephrased as follows: If I can find the first domino, and show that each n^{th} domino will knock down each $(n+1)^{st}$ domino, then we know that all dominoes following that first domino must also fall. At the moment this is rather unclear, so hopefully the following worked examples will help clarify. As afore mentioned, please note that induction is a tool for proving formulas, not for finding formulas. Induction will (typically) not produce a formula for a question, but rather help to show that the formula that you have must hold true.

The general format is the following:

Inductive Hypothesis: the following formula is true for all n greater than u .

Then we show that the formula (Inductive Hypothesis) is true for some value greater than u .

Now we assume the inductive hypothesis is true for some n , and we ask whether the hypothesis is still true for $n+1$, given our assumption that its true for some n .

If we can manage to rearrange the $n+1$ case and substitute in the n case, then it must be true that the formula holds for all n greater than our base case of u .

5.3.1 Example 1

$$\sum_{i=0}^n 2^i = 2^{n+1} - 1$$

Notice that in this example $f(i) = 2^i$. Notice that for each term we're doubling the previous term and adding it on to the current sum.

Before we begin this example, it seems worthwhile to consider some of the importance in this statement. What it literally says is that adding together all the powers of two in a row, will make one short of the next power of two that'll be added. Although a seemingly useless statement, this affirms an absolutely crucial fact in computer science. Computers have been designed to think in binary. Binary uses only 1 and 0 to represent all numbers [I would recommend googling binary or number bases to get a better feel for what this statement says]. A possible concern is that using only 1 and 0, might lead to some gaps in the numbers. However, this example allows you to make certain that using base 2 (binary) counting still accounts for all numbers.

Let us quickly take a look at the $n=3$ case, before we begin, so that we have a better grasp on this formula:

$$\sum_{i=0}^3 2^i = 2^0 + 2^1 + 2^2 + 2^3 = 1 + 2 + 4 + 8 = 15 = 16 - 1 = 2^{3+1} - 1.$$

Ok, now we know that at $n=3$, we have the above summation. Now its time to move to the proof.

First we consider the $n=0$ case:

$$LHS = 2^0 = 1 = 2 - 1 = 2^{(0+1)} - 1 = RHS$$

Now we supposed that $\sum_{i=0}^n 2^i = 2^{n+1} - 1$ is true for some n , and we ask whether it still holds true for $n+1$.

$$\sum_{i=0}^{n+1} 2^i = \sum_{i=0}^n 2^i + 2^{n+1}, \text{ notice that we've just spliced off the last term.}$$

Now we apply the Inductive Hypothesis. When applying the inductive hypothe-

sis, we can replace the part of an expression identical to our Inductive Hypothesis with the expression we're trying to prove. $\sum_{i=0}^n 2^i + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1} = 2 \cdot 2^{n+1} - 1 = 2^{(n+1)+1} - 1 = RHS$, which completes the proof. Since the formula holds true for $n=0$, and the $n+1$ case when true for the n case, the formula will hold true for all $n=0$ or greater.

This is a very powerful statement as we mentioned before.

5.3.2 Example 2

Now we return to the problem that started this all: $1 + 2 + 3 + \dots + (n-1) + n$. Earlier we showed through algebra that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, now let's verify this using our new proof tool of induction.

$$\sum_{i=0}^n i = \frac{n(n+1)}{2}$$

First consider when $n=1$:

$$LHS = 1 = \frac{(1+1) \cdot 1}{2} = RHS, \text{ therefore the formula holds for } n=1.$$

Now we suppose that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ is true for some n , and we ask whether it still holds for $n+1$.

$\sum_{i=1}^{n+1} i = \sum_{i=1}^n i + (n+1) = \frac{n(n+1)}{2} + (n+1)$. Notice that again we applied the same technique of taking off the last term, then plugging in our Inductive Hypothesis. Now we simply need to see if we can alter what we have into the $n+1$ case of our proposed formula. The $n+1$ case of our formula will be $\sum_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2}$, which is simply the formula we started with, except with $n+1$ plugged in for all the n in the original formula.

Now, with our goal in mind, we proceed. $\frac{n(n+1)}{2} + (n+1) = \frac{n(n+1)}{2} + \frac{2 \cdot (n+1)}{2}$, expanding the expression we get: $\frac{n^2+n+2n+2}{2} = \frac{n^2+3n+2}{2}$. The numerator is a quadratic that can either be solved using the famed quadratic formula, or by noticing that it can be factored as $(n+1)(n+2)$.

Putting this back in, we have $\frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2} = \sum_{i=1}^{n+1} i$. This is exactly what we've hoped for! Since we have shown the formula true for $n=1$ and true for all $n+1$ when assumed true for n , the formula will be true for all $n=1$ or greater.

5.3.3 Possible Notation Help

The following simplification is true, and seemed helpful to some people, for these summation based induction proofs. Let's have $g(n) = \sum_{i=1}^n f(i)$. Induction is trying to verify that $g(n+1) = \sum_{i=1}^{n+1} f(i)$, assuming that $g(n) = \sum_{i=1}^n f(i)$

(Inductive Hypothesis). Taking off the last term of the summation in the $n+1$ case, we have: $g(n+1) = \sum_{i=1}^n f(i) + f(n+1)$, now we apply the Inductive Hypothesis, so that $g(n+1) = g(n) + f(n+1)$. If we can show this, then we know that our supposed formula must be true.

5.3.4 Example 3

This last example is more subtle than the previous ones. Although the other ones were less intuitive, this one requires a bit more care while being proved.

$$\sum_{i=1}^n k = n \cdot k$$

In this example, we notice that $f(i) = k$. This means that the function does not depend on i (the index). We should also take note that this is a fairly intuitive statement: a number added to itself n times, is n times the number.

NOTE: the formula we are trying to prove is often called the Inductive Hypothesis.

We begin with the base case, $n=1$:

$$\text{Left-hand side (LHS)} = \sum_{i=1}^1 k = k = 1 \cdot k = \text{Right-hand side (RHS)}.$$

Now we suppose that $\sum_{i=1}^n k = n \cdot k$ is true for some n , and we ask whether it is true for $n+1$.

$\sum_{i=1}^{n+1} k = \sum_{i=1}^n k + k$, which using the Inductive Hypothesis, is also $n \cdot k + k$, which can have the k factored out, making: $(n+1) \cdot k$, which is exactly RHS with $n+1$. This completes the proof. We have found that the formula holds for 1, and for $n+1$ when holding for n . This means that it holds for all $n=1$ or greater.

5.4 Final Words

Before we conclude, we will discuss a little about the uses of induction. Induction is typically used in computer science and or discrete math to prove that certain hypothesized statements are true. Many induction proofs are somewhat similar to the ones seen here, but also there are a number of proofs that involve products of successive terms of a sequence or abstract inductive construction of sets and then using induction on those sets to prove statements about said sets. Needless to say, induction can get a whole ton tougher, but this is supposed to present this material in a somewhat friendly and accessible manner. In addition, it will probably be used a number of times later in this course, so its worthwhile to practice this proof method.

5.5 Questions

Prove the following using induction:

(*)1. $\Sigma 2f(i) = 2\Sigma f(i)$

(*)2. $\sum_{i=0}^{n-1} (2i + 1) = n^2$

(**)3. $\sum_{i=0}^n i^3 = (\sum_{i=0}^n i)^2$

(**)4. $\sum_{i=1}^n (\frac{1}{2})^i = 1 - \sum_{n+1}^{\infty} (\frac{1}{2})^i$. This one is a little less formal than one might desire, as ∞ not very well defined, nor is it clear that ∞ is a natural number. This still serves as practice.

(*)5. $\sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

These are somewhat different than the previous examples (in that they are not summation proofs):

(*)6. Prove that $8^n - 3^n$ is divisible by 5, $\forall n \geq 1$

(*)7. Prove that $n^3 - n$ is always divisible by 6, $\forall n \geq 2$. Hint: factoring might help, also it might be useful to show that this is divisible by 2 and by 3, rather than jump straight to 6

(**)8. Prove that $2^{n+2} + 3^{2n+1}$ is divisible by 7, $\forall n \geq 1$

(**)9. Prove that $11^n - 6$ is divisible by 5, $\forall n \geq 1$

(*)10. Prove $\prod_{i=1}^n \frac{i}{i+1} = \frac{1}{n+1}$. [n.b.: The symbol Π is like Σ except instead of adding progressive terms, successive terms are multiplied together.]

(****)11. Attempt to prove that:

$$\frac{\sum_i a_i}{n} \geq \sqrt[n]{\prod_i a_i}$$

Where a_i is a set of positive numbers. This is true, but challenging.

(**)12. The following is a “proof” by mathematical induction that everyone has the same birthday. Find the flaw in the proof. Explain.

Property P(n): Every member of a set of n distinct people has the same birthday.

Base of induction: Since a set of one person has only one birthday, so P(1) is true.

Inductive step: Assume P(k) is true for a positive integer k, we will show that P(k+1) is also true.

For a set $A = \{a_1, a_2, a_3, \dots, a_{k+1}\}$ of k+1 distinct people, consider two subsets $B = \{a_1, a_2, a_3, \dots, a_k\}$ and $C = \{a_2, a_3, \dots, a_{k+1}\}$ each with k distinct people,

obtained respectively by removing the last and first person from the set A. By the inductive assumption, every member of set B has the same birthday x , and every member of set C has the same birthday y . Since the two sets B and C have a_2, a_3, \dots, a_k in common, the two birthdays x and y must be the same. As a result, every member of the set A has the same birthday and we have shown $P(k+1)$ is true based on the inductive hypothesis that $P(k)$ is true.

Chapter 6

Proof by contradiction

This is another method of proof. Again, please do not try to use this to conjure up a formula, this is only useful for proving validity, although in a different way than proofs by induction.

Consider a proposed theorem (generalized statement in math) that claims that something is true. Proof by contradiction works by assuming that its not true. If, while trying to prove that its not true, you come across something that is mathematically false or contradicts an initial assumption, then your assumption that the theorem is wrong, is itself wrong, so the theorem is right. By going through this, you have disproved the alternative option, and have thus solidified the proposed theorem. You have proven that the statement cannot be false.

6.1 Example

In chapter 1, we asked whether $\sqrt{2}$ was a rational number. Although its reasonable to assume that its not, how would we know for certain? This is an opportunity for proof by contradiction.

We begin by assuming that $\sqrt{2} = \frac{p}{q}$, where p,q are relatively prime natural numbers. We now consider the square of the initial relation:

$$\begin{aligned}\sqrt{2} &= \frac{p}{q} \\ 2 &= \frac{p^2}{q^2} \\ p^2 &= 2q^2\end{aligned}$$

Now we consider this statement. It says that some number is equal to twice another number. This implies that p^2 is an even number. Since p^2 is even, p must be even. This is perfectly fine (for now). Since p is even, we can write it as $2k$, where k is a natural number. Now we start over again, except replacing p with $2k$. So we have:

$$\begin{aligned}\sqrt{2} &= \frac{2k}{q} \\ 2 &= \frac{(2k)^2}{q^2} \\ q^2 &= \frac{4k^2}{2} \\ q^2 &= 2k^2\end{aligned}$$

We notice that the right side is again twice a number, which means q^2 is even, which means q is even and can be expressed as $2w$. This means that if $\sqrt{2}$ can be expressed as a rational number, then its equal to $\frac{p}{q} = \frac{2k}{2w}$, but this implies that p, q have a common factor of 2, which means that they're not relatively prime as we demanded initially. This is our contradiction, which means that $\sqrt{2}$ cannot be a rational number. This completes the proof.

We only give one example because this is a relatively straight forward proof method that just requires practice and a little intuition.

6.2 Questions

(*)1. Prove $\sqrt{3}$ is irrational

(**)2. Prove the following set of statements:

the product of two positive numbers is positive
the product of two negative numbers is positive
the produce of a positive and a negative number is negative

(**)3. Prove the following statements:

the product of two even numbers is an even number
the product of two odd numbers is an odd number
the product of an even number and an odd number is an even number

(*)4. Prove that the difference between an irrational number and a rational number must be irrational.

(*)5. Prove that if a number m is rational, and the product mn is irrational, then n must be irrational.

Chapter 7

Proof by pigeon holing

Consider your classroom. There are maybe about 30 students in the room. Your mind begins to wander into new domains, and you ask yourself: what're the odds that any two people in this room have the same birthday? Or simpler, what's the maximum number of people needed to guarantee that not everyone in the room has a unique birthday?

Enter pigeon holing. The principle of pigeon holing is that you have a certain number of possible outcomes, but once you have more trials than possible outcomes, then you must have some repetitions. This is a proof of the minimum number of trials needed to ensure a collision or overlap. Its not too useful for verifying formulas, but rather for bounding formulas or expressions.

7.1 Examples

7.1.1 Example 1

Consider the maximum number of coin flips that need to be made before a repeat face is shown.

On the first flip we get either a heads or a tails. Best case we accidentally get the same face twice in a row and that's the end, only one flip. Worst case, on the next flip we land the other face, and now all faces have been shown. On the third flip we get one side and it must be a repeat, which means the most number of flips is a measly three.

7.1.2 Example 2

Consider the birthday problem we began this chapter with. There are only 365 (or 366) days in a year. Each person has some birthday. This means we can apply the same logic as in the previous example, but now we need to repeat it roughly 365 times. This means that the least people we can trap in a room

and know for certain that there is a repeat birthday is 367 people (366 and the certain repeater).

7.2 Questions

(*)1. How many rolls of a 6-sided die are needed to ensure that a repeated roll occurs?

(*)2. How many rolls of an n -sided die are needed to ensure that a repeated roll occurs?

(**)3. How many real numbers do we need to pick between 0 and 1 so that we have an ensured repeat?